# Derivatives as Rate of Change

$y = f(x)$ (given)

$$y' = f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$\Delta y \rightarrow$ change in $y$

$\Delta x \rightarrow$ change in $x$

Some notions associated to **Motion**:

$s(t)$ = position of an object at time $t$.

$v(t) = s'(t)$ = velocity (not average) of an object at time $t$.

$|v(t)|$ = speed of an object at time $t$.

$a(t) = v'(t) = s''(t)$ = acceleration of an object at time $t$.

## Moving forward & backward:

Moving forward $\Leftrightarrow$ with increasing time $s(t)$ increases.
$\Updownarrow$
velocity is positive.

Moving backward $\Leftrightarrow$ with increasing time $s(t)$ decreases.
$\Updownarrow$
velocity is negative.

## Increasing & Decreasing Velocity:

Increasing velocity $\Leftrightarrow$ +ve accelaration

Decreasing velocity $\Leftrightarrow$ −ve accelaration

## Speeding Up & Speeding Down.

Speeding Up $\Leftrightarrow$ Velocity & Accelaration have the same sign.

Speeding Down $\Leftrightarrow$ Velocity & Accelaration have the opposite sign.

<u>Example</u>: Position of an object is given by $s(t) = \dfrac{4t}{t^2+4}$. $(t \geq 0)$.

- When the object is moving forward/backward?
- When the object's velocity increasing/decreasing?
- When the object is speeding up or down?

– $s(t) = \dfrac{4t}{t^2+4}$, $t \geq 0$

$$s'(t) = \frac{(t^2+4)[4t]' - 4t[t^2+4]'}{(t^2+4)^2}$$

$$= \frac{(t^2+4)(4) - 4t(2t)}{(t^2+4)^2}$$

$$= \frac{4[(t^2+4) - 2t^2]}{(t^2+4)^2}$$

$$= \frac{4[-t^2+4]}{(t^2+4)^2}$$

$$= \frac{\overset{-ve}{-4}(t^2-4)}{\underbrace{(t^2+4)^2}_{+ve}}$$

it's neither +ve or −ve } so we need conditions.

Now, $t^2 - 4 = \underbrace{(t+2)}_{+ve}(t-2)$ $\begin{cases} > 0 & \text{if } t > 2 \\ < 0 & \text{if } t < 2 \end{cases}$

$$\boxed{\text{So, } s'(t) < 0 \text{ if } t > 2 \;\&\; s'(t) > 0 \text{ if } t < 2.}$$

ie, the object is moving forward when $0 \leq t < 2$ &
the object is moving backward when $t > 2$.

$$- \quad s''(t) = \left[ - \frac{4(t^2-4)}{(t^2+4)^2} \right]'$$

$$= \frac{-4\left[ (t^2+4)^2 \cdot (t^2-4)' - (t^2-4)\left[(t^2+4)^2\right]' \right]}{\left[(t^2+4)^2\right]^2}$$

$$= \frac{-4\left[ (t^2+4)^2 \cdot 2t - (t^2-4) \cdot 2(t^2+4) \cdot 2t \right]}{(t^2+4)^4}$$

$$= \frac{-4 \cdot 2t (t^2+4)\left[ (t^2+4) - 2(t^2-4) \right]}{(t^2+4)^4}$$

$$= \frac{-8t\left[-t^2+12\right]}{(t^2+4)^3}$$

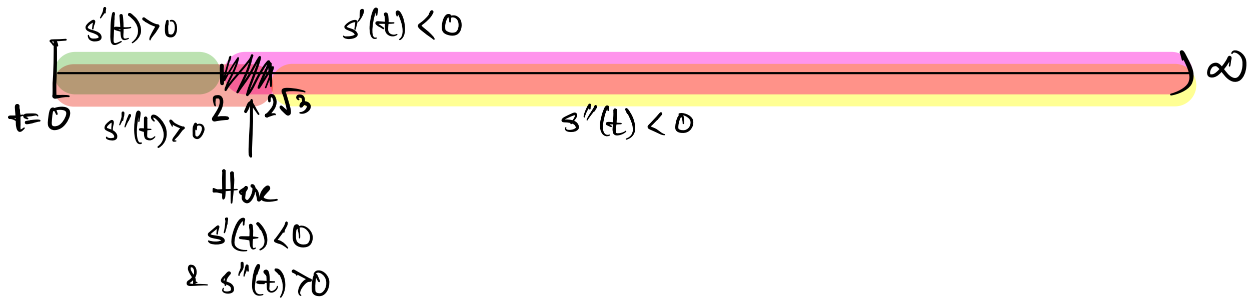
$$= \frac{\overset{+ve}{8t}\left[t^2-12\right]}{\underset{+ve}{(t^2+4)^3}}$$

as $t \geq 0 \Rightarrow t^2+4 > 0 \Rightarrow (t^2+4)^3 > 0$

$$\text{Now, } t^2 - 12 = t^2 - (\sqrt{12})^2 = \underset{+ve}{(t+\sqrt{12})}(t-\sqrt{12}) \begin{cases} > 0, & t > \sqrt{12} = 2\sqrt{3} \\ < 0, & t < \sqrt{12} = 2\sqrt{3} \end{cases}$$

So, $s''(t) > 0$ when $t > 2\sqrt{3}$ $\Rightarrow$ increasing velocity for $t > 2\sqrt{3}$
$s''(t) < 0$ when $t < 2\sqrt{3}$. $\Rightarrow$ decreasing velocity for $t < 2\sqrt{3}$.

- Chart to decide speeding up & down.




So, the object is speeding up on $0<t<2$ & $2\sqrt{3}<t<\infty$

& speeding down on $2<t<2\sqrt{3}$.